A Sub-optimal Joint Subcarrier and Power Allocation Algorithm for Multiuser OFDM

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Abstract— **This paper investigates subcarrier and power allocation in multiuser OFDM. The aim is to maximize the overall rate while achieving proportional fairness amongst users under a total power constraint. Achieving the optimal solution is computationally demanding thereby necessitating the use of sub-optimal techniques. Existing sub-optimal techniques either use fixed power allocation and perform only subcarrier allocation or handle subcarrier and power allocation separately. In this paper, we propose an algorithm that performs joint subcarrier and power allocation. Simulation results are shown to compare the performance of the proposed algorithm with that of existing algorithms.**

Index Terms— **Water-filling, power allocation, subcarrier allocation, multiuser OFDM, OFDMA.**

I. Introduction

RESOURCE allocation techniques for multiuser Orthogonal Frequency Division Multiplexing (OFDM) are of two types: fixed [7] and dynamic [1] [2] [3] [4]. Fixed resource allocation techniques fail to exploit multiuser diversity resulting in poor system performance. On the other hand, dynamic resource allocation techniques allocate resources (subcarriers and time slots) taking into account the users' current channel conditions.

In this paper, we focus on rate adaptive dynamic resource allocation [2] [3] [4]. In rate adaptive resource allocation, subcarrier and power allocation are performed to maximize the overall rate while achieving proportional fairness amongst users under a total power constraint. Though proportional fairness amongst users is achieved in [3], the frequency selective nature of a user's channel is ignored by allocating power uniformly across all subcarriers belonging to a particular user. The algorithm proposed in [4] adopts a two step approach. In the first step, the algorithm outlined in [3] is employed for subcarrier allocation. In the second step, [4] takes into account the frequency selective nature of a user's channel through the use of water-filling [5] during power allocation to each user. The proposed algorithm takes into account the frequency selective nature of users' channels and performs joint subcarrier and power allocation thereby avoiding the two step approach outlined in [4].

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II. Subcarrier and Power Allocation in Multiuser OFDM

In this paper, we assume that the base station has knowledge of the channels of all the users in the system. Since channel conditions vary over a period of time, this information must be updated periodically with the help of feedback channels.

In the rate adaptive technique under consideration, subcarrier and power allocation have to be carried out jointly to achieve the optimal solution. For the sake of simplicity, each subcarrier is allocated to only one user at any instant of time. The optimization problem can be formulated as,

$$\max_{P_{k,n},A_k} \sum_{k=1}^{K} \sum_{n \in A_k} \log_2(1 + P_{k,n}\gamma_{k,n}) \tag{1}$$

$$\begin{aligned} &\text{subject to } \sum_{n=1}^{N}\sum_{k=1}^{K} P_{k,n} \le P_{total} \\ &P_{k,n} \ge 0 \text{ for all } k, n \\ &A_1, A_2, ..., A_K \text{ are all disjoint} \\ &A_1 \cup A_2 \cup ... \cup A_K = \{1, 2, ..., N\} \\ &R_1 : R_2 : .... : R_K = \alpha_1 : \alpha_2 : .... : \alpha_K \end{aligned}$$

where K is the total number of users, N is the total number of subcarriers, P_{total} is the overall available power, $P_{k,n}$ is the power allocated to the k^{th} user in the n^{th} subcarrier, $\gamma_{k,n} = \frac{|H_{k,n}|^2}{N_0 \frac{B}{N}}$ is the channel gain to noise power ratio for the k^{th} user in the n^{th} subcarrier, $H_{k,n}$ is the channel gain for the k^{th} user in the n^{th} subcarrier, N_0 is the power spectral density (PSD) of additive white Gaussian noise (AWGN), B is the overall available bandwidth, and A_k is the set of all subcarriers allocated to the k^{th} user. $\{\alpha_1, \alpha_2, ..., \alpha_K\}$ is a set of predetermined constants to ensure proportional fairness amongst users. R_k is the k^{th} user's rate defined as

$$R_k = \sum_{n \in A_k} \log_2(1 + P_{k,n}\gamma_{k,n}) \tag{2}$$

III. Proposed Solution

In this section, we propose a sub-optimal solution to the optimization problem in (1). Each of the N subcarriers is to be allocated to one of K users. In addition, the power allocated to each of the K users is to be optimized. This means that $N+K$ parameters need to be optimized to achieve the optimal solution. Power allocation amongst subcarriers belonging to a particular user is achieved through water-filling. Achieving the optimal solution is computationally demanding thereby necessitating the use of sub-optimal techniques [3]

[4]. The algorithm proposed in [3] simplifies the optimization problem in (1) into one that has N optimization parameters by assuming equal power allocation to all subcarriers, i.e,

$$P_{k,n} = \begin{cases} \frac{P_{total}}{N} & \text{for } n \in A_k \\ 0 & \text{otherwise} \end{cases} \quad (3)$$

for all $n = 1, 2, \cdots, N$ and $k = 1, 2, \cdots, K$. Since the power allocated to each subcarrier is fixed, optimization now involves assigning the N subcarriers to K users. The algorithm proposed in [4] uses a two step approach to optimize $N + K$ parameters. In the first step, the strategy outlined in [3] is used to assign subcarriers to users. In the next step, the power allocated to the K users is determined by solving a non-linear equation. Power allocation amongst subcarriers belonging to a particular user is achieved through water-filling.

In the proposed solution, optimization of the $N + K$ parameters is carried out by alternating between subcarrier and power allocation. As in [4], water-filling is used for each user. However, unlike [4], water-filling for each user plays a crucial role in deciding the subcarrier allocation. When a subcarrier is allocated to a user, the power allocated to the user is incremented by $\frac{P_{total}}{N}$, i.e., the power allocated to each user is proportional to the number of subcarriers currently allocated to the user. The user's rate is also updated assuming that water-filling is used. This updated rate information is used in the allocation of the remaining subcarriers. Thus, the gain from water-filling is seen in the subcarrier allocation stage by all the users resulting in higher user rates. It is interesting to note that, unlike [4], the proposed algorithm can easily incorporate practical power allocation algorithms such as [6] instead of water-filling.

The joint subcarrier and power allocation strategy is as follows.

1. Initialize $A = \{1, 2, 3, ...N\}$
2. $\forall\, k = 1$ to K, $A_k = \phi, P_k = 0$
3. $\forall\, k = 1$ to K,
   (a) $\gamma_k = \max_n \gamma_{k,n}$ for $n \in A$
   (b) $A_k = A_k \cup \{n\}$, $P_k = P_k + \frac{P_{total}}{N}$
   (c) $R_k = \log_2(1 + P_k \gamma_k)$
   (d) $A = A - \{n\}$
4. While $A \neq \phi$,
   (a) find i such that $\frac{R_i}{\alpha_i} \leq \frac{R_k}{\alpha_k}\ \forall\, k, i = 1$ to K
   (b) for the above i, find n such that
   $$\gamma_{i,n} \geq \gamma_{i,m}\ \forall\, n, m \in A$$
   (c) $A_i = A_i \cup \{n\}$, $P_i = P_i + \frac{P_{total}}{N}$
   (d) $A = A - \{n\}$
   (e) $R_i = \sum_{n \in A_i} log_2(1 + P_{i,n}\gamma_{i,n})$ where $P_{i,n} = \left(\gamma - \frac{1}{\gamma_{i,n}}\right)^+$ and $\sum_{n \in A_i} P_{i,n} = P_i$

The $f(x) = (x)^+$ operator indicates that $f(x) = 0$ when $x < 0$ and $f(x) = x$ when $x \geq 0$.

The subcarrier and power allocation strategy described above follows the strategy used in [3] except for the rate update equation 4(e). While [3] uses uniform power allocation across all subcarriers belonging to a user, our algorithm uses water-filling for each user. The rate update equation used is [3]

$$R_i = \sum_{n \in A_i} \log_2\left(1 + \frac{P_{total}}{N}\gamma_{i,n}\right). \quad (4)$$

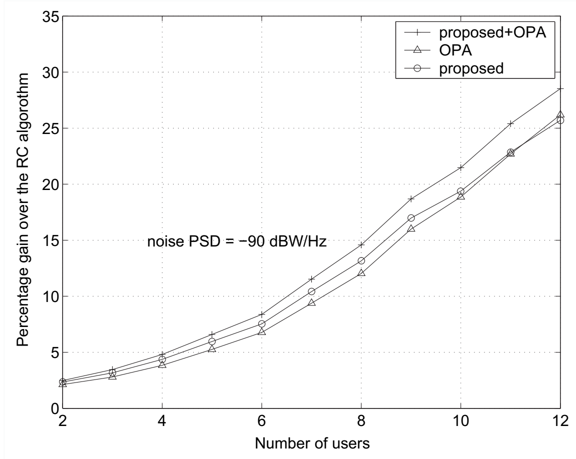

Fig. 1. Gain over the RC algorithm vs. Number of users

The proposed algorithm requires water-filling to be performed $N - K$ times in addition to the computation required for the algorithm in [3]. The algorithm in [4] requires (a) the computation required for the algorithm in [3], (b) setting up and solving the non-linear equations for K variables, and (c) water-filling to be performed K times.

Throughout the simulations, we have used water-filling in step 4(e) after a subcarrier is allocated to a user. This is for the purpose of evaluating the performance of the proposed algorithm against existing algorithms [3] [4]. However, in practice, power allocation algorithms such as [6] can be used during step 4(e) of the proposed algorithm.

IV. Simulation Results

Simulation results are shown for an $N = 256$ subcarrier multiuser OFDM system with bandwidth 1 MHz. Each user is assumed to have a 6-tap sample-spaced multipath channel with each tap experiencing independent Rayleigh fading. The tap energies are assumed to decay exponentially as in [4], i.e.,

$$E[|h_{l,k}|^2] = g_k e^{-(l-1)} \text{ for } l = 1, 2, .., 6,\ k = 1, 2, .., K. \quad (5)$$

where $h_{l,k}$ is the l^{th} time domain tap for the k^{th} user and g_k is the tap energy of the first tap for the k^{th} user. The total power available at the transmitter $P_{total} = 1$ Watt. Let $\alpha_1 : \alpha_2 : \alpha_3 : ... : \alpha_K = 1 : 1 : 1 : ... : 1$ so that the overall rate is maximized while trying to achieve equal rate for all users.

The spectral efficiency that can be achieved by the proposed algorithm for a given proportional fairness constraint is compared with that of the algorithms in [3] (referred to as the RC algorithm) and [4] (referred to as the OPA algorithm). We define spectral efficiency as follows,

$$\text{Spectral efficiency} = \left(\min_k R_k\right) * (S) * \left(\frac{K}{B}\right)$$

where S is the OFDM symbol rate in symbols per second. Comparison is also made with the spectral efficiency achieved when power allocation is performed with the OPA algorithm after all subcarriers have been allocated with the proposed

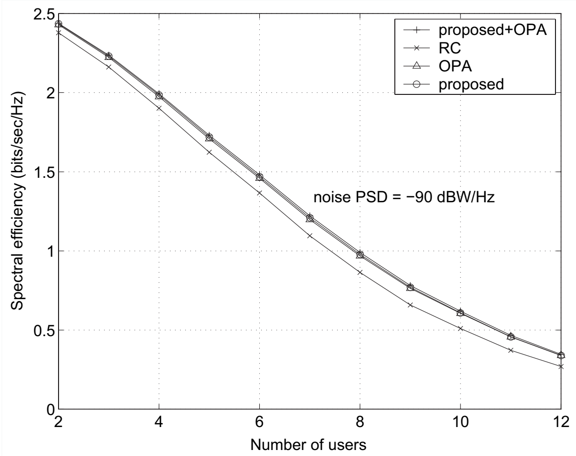

Fig. 2. Spectral efficiency vs. Number of users

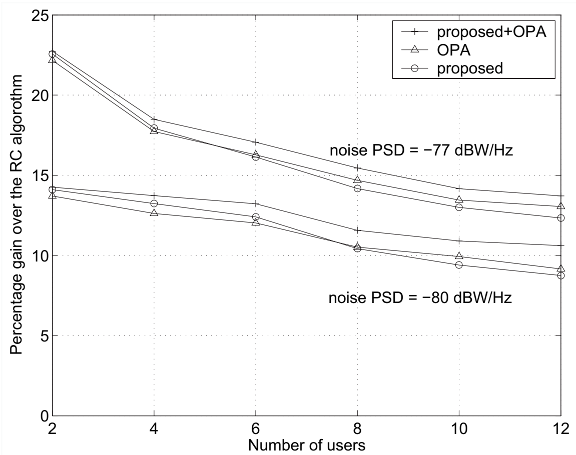

Fig. 3. Gain over the RC algorithm vs. Number of users

algorithm (referred to as the proposed+OPA algorithm). The performance of the algorithms is evaluated when the PSD of AWGN N_0 is -90 dBW/Hz and

$$g_k = -2(k-1) \text{ dB for } k = 1, 2, ..., K \quad (6)$$

Figures 1 and 2 show the percentage gain in spectral efficiency over the RC algorithm and the spectral efficiency against the number of users in the system, respectively. It is evident that the proposed algorithm performs slightly better than the OPA algorithm when the number of users is small. As the number of users increases, there is a slight degradation in performance of the proposed algorithm relative to the OPA algorithm. This is because of a small deviation in enforcing the proportional fairness using our algorithm when the number of users is large. The same effect can be observed even with the RC algorithm [4]. However, the proposed+OPA algorithm achieves proportional fairness exactly while using our improved subcarrier allocation.

Figure 3 shows the percentage gain in spectral efficiency over the RC algorithm when all users in the system experience similar channel conditions, i.e,

$$g_k = 1 \text{ for } k = 1, 2, ..., K \quad (7)$$

The gain achieved over the RC algorithm is evaluated when the PSD of AWGN N_0 is -80 dBW/Hz and -77 dBW/Hz. When the PSD of AWGN is higher (-77 dBW/Hz), the gain achieved over the RC algorithm is higher because water-filling yields larger gains at low SNRs [8].

V. Conclusions

In this paper, we proposed an algorithm that performs joint subcarrier and power allocation while taking into account the frequency selective nature of users' channels. Simulation results have shown that the performance of the proposed algorithm is comparable to that of the OPA algorithm. However, unlike the OPA algorithm, the proposed algorithm can readily use practical power allocation algorithms such as [6] instead of water-filling.

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